7 More Graphing Rational Equations

Warmup

1. $(x^3 - 1) \div (x - 1)$

   $x^2 + x + 1$

2. $(y^3 + 27) \div (y + 3)$

   $y^2 - 3y + 9$

3. $(64x^3 + 27) \div (4x + 3)$

   $16x^2 - 12x + 9$

4. $(27x^3 + 8) \div (3x + 2)$

   $9x^2 - 6x + 4$

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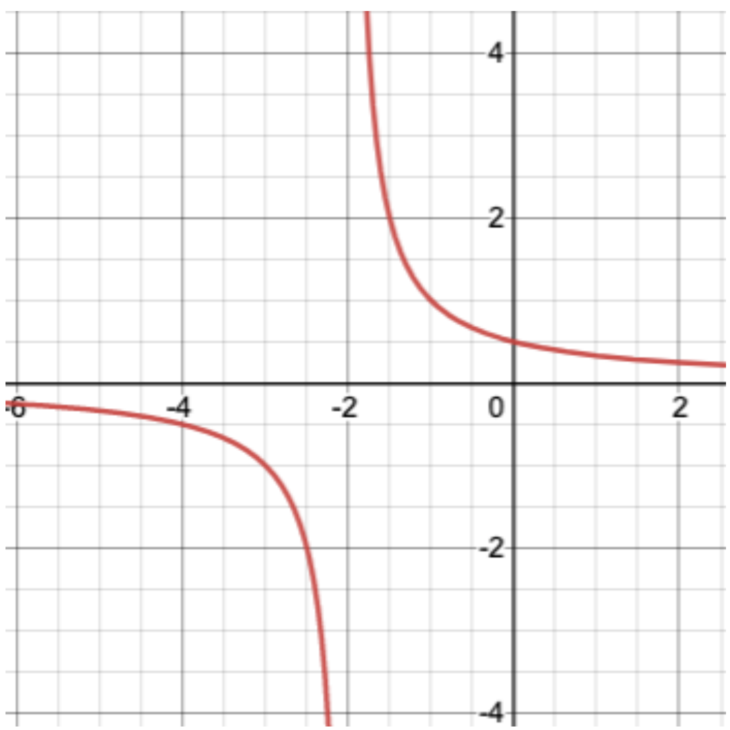
Domain

$$f(x) = \frac{1}{x+2}$$

Vertical or horizontal asymptotes?

Vertical (x = -2), Horizontal (y = 0)



What's the behavior of the graph as it approaches the vertical asymptote from the right/left?

$$\lim_{x \to -2^-} f(x) = -\infty$$ From left

$$\lim_{x \to -2^+} f(x) = +\infty$$ From right

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Domain

$$f(x) = \frac{1}{(x+2)(x-3)}$$

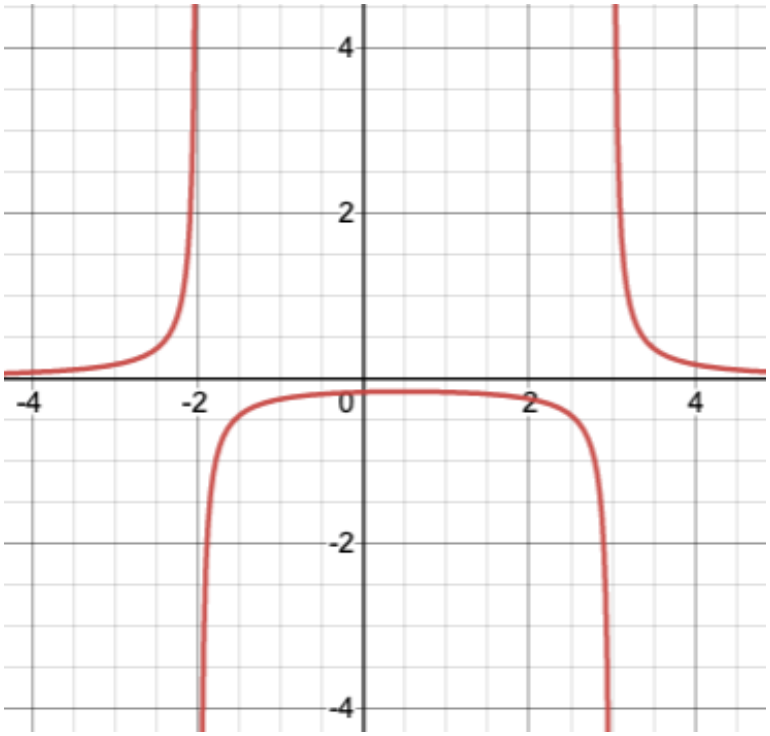
What's the behavior of the graph as it approaches the vertical asymptote from right/left?

$$\lim_{x \to -2^-} f(x) = +\infty$$

$$\lim_{x \to -2^+} f(x) = -\infty$$

$$\lim_{x \to 3^-} f(x) = -\infty$$

$$\lim_{x \to 3^+} f(x) = +\infty$$



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Domain

$$f(x) = \frac{1}{x+2}$$

$\lim_{x \to -2^-} f(x) = -\infty$ From left

$\lim_{x \to -2^+} f(x) = +\infty$ From right

Practice - Describe the behavior at values of x not in domain.

1. $f(x) = \frac{-1}{x-2}$

   $\lim_{x \to 2^+} f(x) = -\infty$

   $\lim_{x \to 2^-} f(x) = +\infty$

2. $f(x) = \frac{1}{x^2 - 9}$

   $\lim_{x \to 3^+} f(x) = +\infty$

   $\lim_{x \to 3^-} f(x) = -\infty$

   $\lim_{x \to -3^+} f(x) = -\infty$

   $\lim_{x \to -3^-} f(x) = +\infty$

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Domain

$$f(x) = \frac{1}{x}$$

This is the original function that we will be transforming.

What is h(x) in terms of f(x)?

$$h(x) = \frac{1}{x-2} \quad = f(x-2)$$

$$h(x) = \frac{3}{x+5} \quad = 3 \cdot f(x+5)$$

$$h(x) = \frac{-1}{x-2} + 3 \quad = -f(x-2) + 3$$

$$h(x) = \frac{3x-7}{x-2}$$

$$h(x) = 3 - \frac{1}{x-2} = 3 - f(x-2)$$

Let's look at the behavior near the vertical asymptotes.

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Behavior around asymptotes

$$h(x) = \frac{1}{x-2} \quad = f(x-2)$$

$$\lim_{x \to 2^+} h(x) = +\infty$$

$$\lim_{x \to 2^-} h(x) = -\infty$$

$$h(x) = \frac{3}{x+5} \quad = 3 \cdot f(x+5)$$

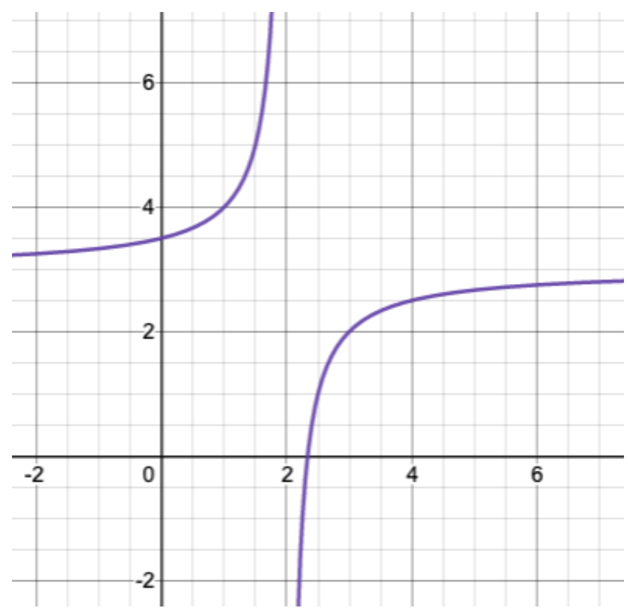
$$\lim_{x \to -5^+} h(x) = +\infty$$

$$\lim_{x \to -5^-} h(x) = -\infty$$

$$h(x) = \frac{-1}{x-2} + 3 \quad = -f(x-2) + 3$$

$$\lim_{x \to 2^+} h(x) = -\infty$$

$$\lim_{x \to 2^-} h(x) = +\infty$$



7 More Graphing Rational Equations

Behavior around asymptotes

$$h(x) = \frac{3}{x+5} \quad = 3 \cdot f(x+5)$$

$$\lim_{x \to -5^+} h(x) = +\infty$$

$$\lim_{x \to -5^-} h(x) = -\infty$$

$$h(x) = \frac{-1}{x-2} + 3 \quad = -f(x-2) + 3$$

$$\lim_{x \to 2^+} h(x) = -\infty$$

$$\lim_{x \to 2^-} h(x) = +\infty$$

Practice - Transform into f(x) = 1/x and describe behavior

1. $h(x) = \dfrac{-3}{x+7}$

$$h(x) = -3f(x+7)$$

$$\lim_{x \to -7^+} h(x) = -\infty$$

$$\lim_{x \to -7^-} h(x) = +\infty$$

2. $h(x) = \dfrac{-4x+3}{x-2}$

$$h(x) = -5f(x-2) - 4$$

$$\lim_{x \to 2^+} h(x) = -\infty$$

$$\lim_{x \to 2^-} h(x) = +\infty$$

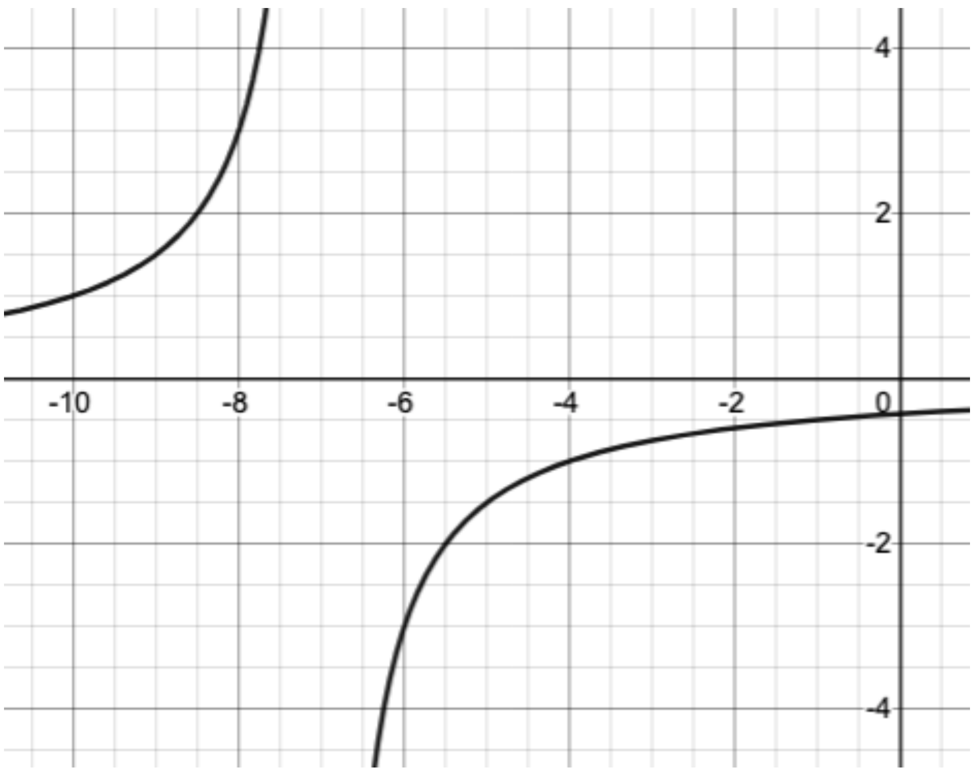
7 More Graphing Rational Equations

Let's look at it graphically

1. $h(x) = \dfrac{-3}{x+7}$

$h(x) = -3f(x+7)$

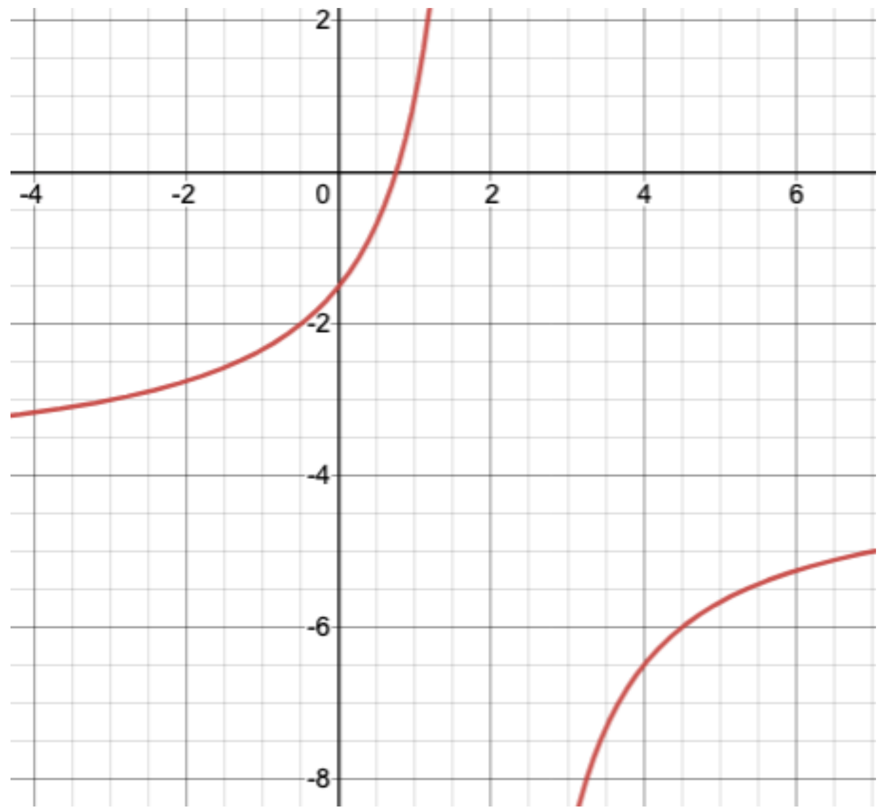
$\lim_{x \to -7^+} h(x) = -\infty$ $\lim_{x \to -7^-} h(x) = +\infty$



2. $h(x) = \dfrac{-4x+3}{x-2}$

$h(x) = -5f(x-2) - 4$

$\lim_{x \to 2^+} h(x) = -\infty$ $\lim_{x \to 2^-} h(x) = +\infty$



7 More Graphing Rational Equations

Asymptotes and Intercepts

Find the asymptotes and intercepts of the function.
Sketch the graph.

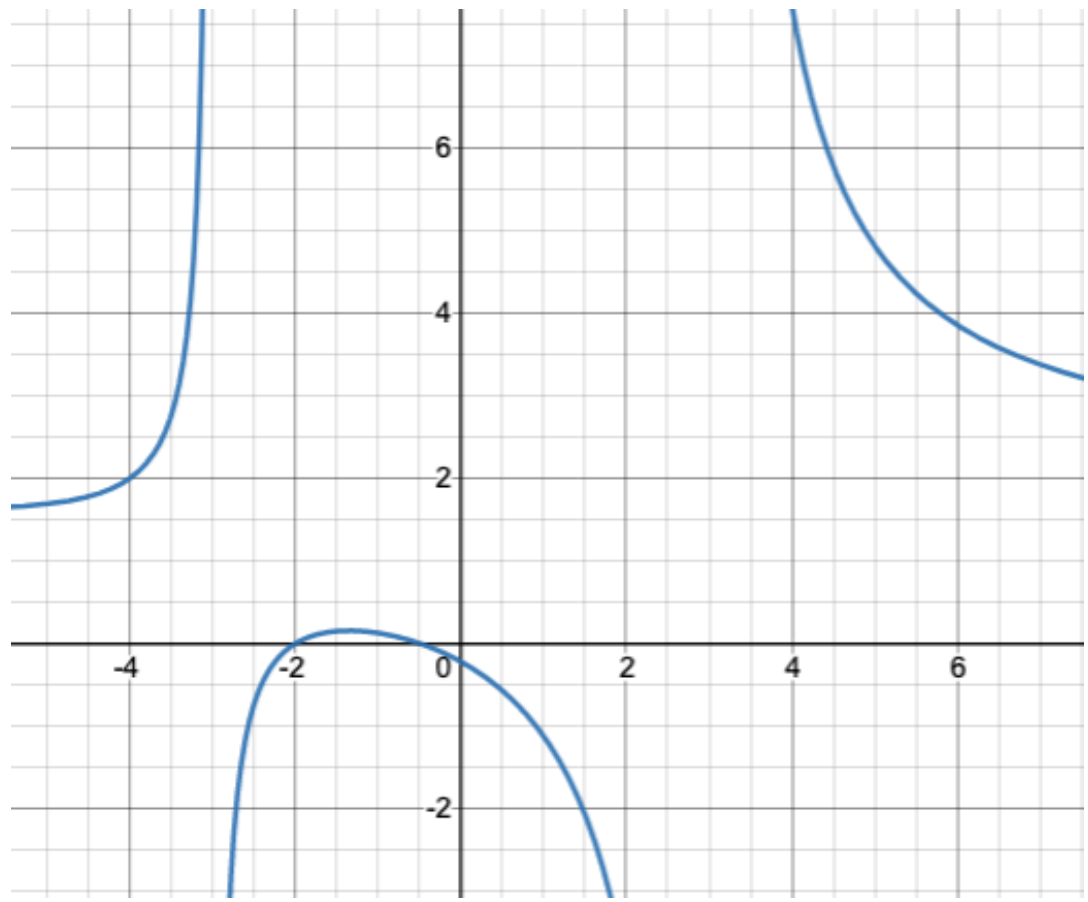
$$f(x) = \frac{2x^2 + 5x + 2}{x^2 - 9}$$

Vertical asymptotes
  x = 3, x = -3

Horizontal asymptotes
  y = 2

x-intercepts
  x = -2, x = $-\frac{1}{2}$

y-intercepts
  y = $-\frac{2}{9}$



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Asymptotes and Intercepts

What about the horizontal asymptote of:

$$f(x) = \frac{2x^2 + 3x + 4}{3x^3 + 6x^2 - x + 7}$$

$$\lim_{x \to \infty} f(x) = 0$$

So horizontal asymptote y = 0

7 More Graphing Rational Equations

Asymptotes and Intercepts

$$f(x) = \frac{2x^3 + 3x^2 - 4x - 6}{x^4 - 16}$$

Vertical asymptotes

$x = 2, -2$

Horizontal asymptotes

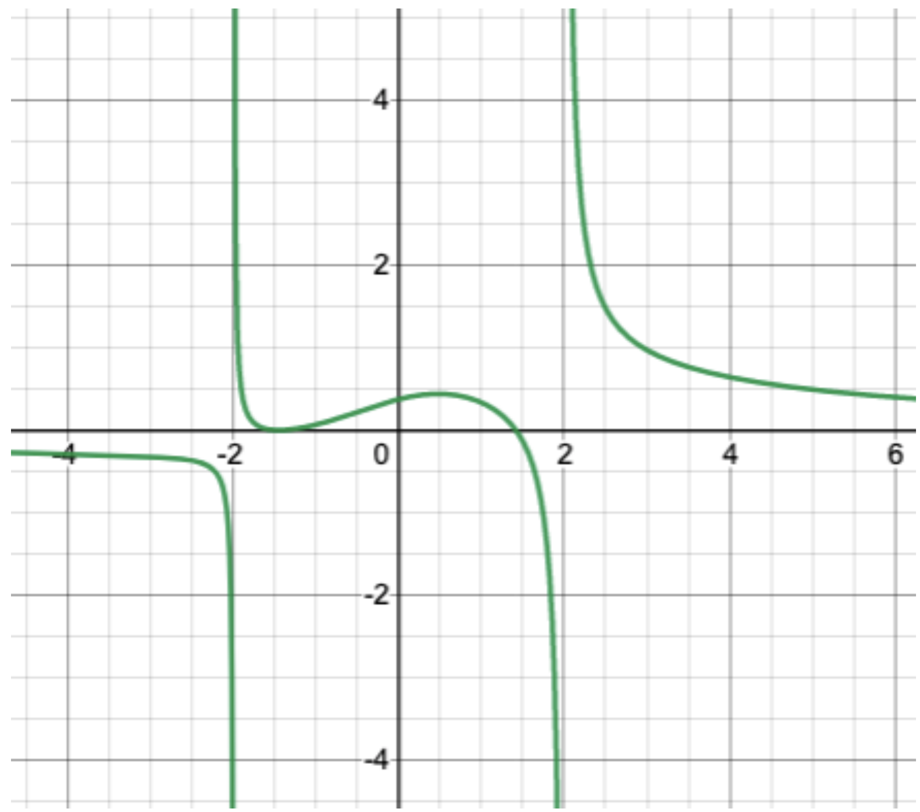
$y = 0$

x-intercepts

$x = -\frac{3}{2}, \pm\sqrt{2}$

y-intercepts

$y = \frac{3}{8}$



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Finding Asymptotes

Vertical asymptotes occur at the zeros of the denominator provided that the zeros are not also zeros of the numerator of equal or greater multiplicity.

$$f(x) = \frac{2x+1}{x+3} \qquad f(x) = \frac{2x+1}{(2x+1)(x-3)} \qquad f(x) = \frac{x+1}{(x+1)^2}$$

Asymptotes: $x = -3$ $\qquad$ $x = 3$ $\qquad$ $x = -1$

Hole at x = $-\frac{1}{2}$

7 More Graphing Rational Equations

Finding Asymptotes

Find x-intercepts, y-intercepts, and all asymptotes

1. $f(x) = \dfrac{2x^2 - 1}{x^2 - 4}$ $x - int : \pm\dfrac{\sqrt{2}}{2}$ $y - int : \dfrac{1}{4}$ Asymptotes: $x = \pm 2,\ y = 2$

2. $f(x) = \dfrac{2x - 1}{4x^2 - 1}$ $x - int :\ none$ $y - int :\ 1$ Asymptotes: $x = -\frac{1}{2},\ y = 0$ Hole at $x = \frac{1}{2}$

7 More Graphing Rational Equations

Graphing

$$f(x) = \frac{-1}{6x^2 + x - 1}$$

Vertical asymptotes

$x = -\frac{1}{2}, x = \frac{1}{3}$

x-intercepts

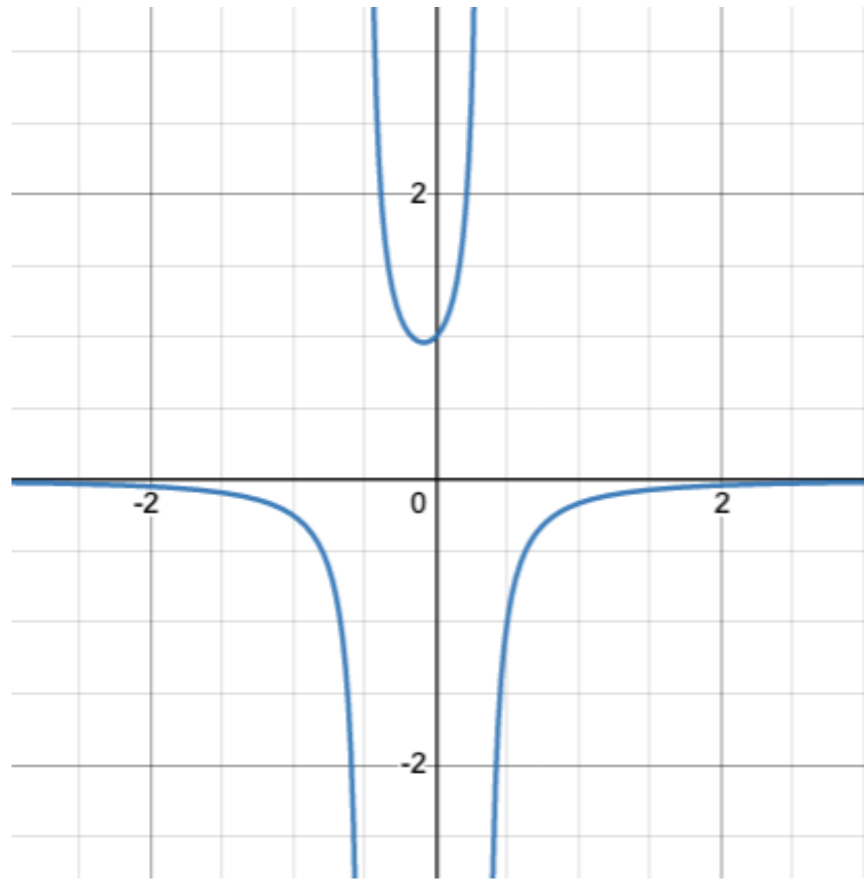
none

y-intercepts

$y = 1$

Horizontal asymptotes

$y = 0$



Behavior near vertical asymptotes

$$-\frac{1}{2}^{+} = \infty, -\frac{1}{2}^{-} = -\infty, \frac{1}{3}^{+} = -\infty, \frac{1}{3}^{+} = \infty$$